On the dependence of investor's probability of default on climate transition scenarios

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Abstract

Climate risk brings about a new type of financial risk that standard approaches to risk management are not adequate to handle. Amidst the growing concern about climate change, financial supervisors and risk managers are concerned with the risk of a disorderly low-carbon transition. We develop a model to compute i) the valuation adjustment of corporate bonds, depending both on climate transition risk scenarios and on companies' shares of revenues across low/high-carbon activities, and ii) the corresponding adjustments of an investor's Expected Shortfall and probability of default. Implications for central banks' climate financial risk management include that climate stress test exercises should allow for a wide enough set of scenarios in order to limit the underestimation of losses.

Keywords: climate transition risk, climate policy scenarios, probability of default, corporate bonds, Expected Shortfall, climate financial risk assessment, risk management.

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1. Introduction

Central banks and financial regulators are increasingly concerned about the impact of climate change on financial stability (Carney, 2015; ECB, 2019; FSB, 2020), and recommended companies and investors to assess their exposure to climate risks (TCFD, 2017; NGFS, 2019). In particular, financial supervisors worry about scenarios of climate transition risk, in which the delay in climate policy or regulation, and their impact, cannot be fully anticipated by investors. This, in turn, creates the conditions for a disorderly low-carbon transition (NGFS, 2019) with implications for asset price volatility and financial instability (Gros et al., 2016). Their concerns are supported by a stream of research in climate finance (see e.g. Dietz et al. (2016); Battiston et al. (2017)).

Recognizing that climate transition risk represents a new and material risk for investors, several international financial institutions joined the Network for Greening the Financial System (NGFS) with the aim to support financial actors in assessing their exposure to climate-related financial risks. Recently, the NGFS has developed a set of scenarios of the low-carbon transition relevant for financial actors' climate stress-test exercises (NGFS, 2020). Among financial supervisors and institutions there is an intuition that, in a disorderly transition, issuers with a larger (lower) exposure to low-carbon activities may be less (more) risky. As a result, a portfolio with larger exposure to issuers with low-carbon (high-carbon) activities could be less (more) risky (Depres and Hiebert, 2020; Allen, 2020; Grippa et al., 2020).

However, to the best of our knowledge, there has been no model describing analytically or by means of numerical computations, i) the valuation adjustment of a corporate bond, depending both on climate transition risk scenarios and on companies' shares of revenues across low/high-carbon activities, and ii) the corresponding adjustments in an investor's Expected Shortfall and probability of default.

A fundamental difficulty for assessing climate change risk comes from the fact that it is forward-looking (magnitude of future impacts cannot be calculated based on backward-looking information), it is endogenous (i.e. the realization of adverse scenarios depend on risk perception and reaction of agents) and involves multiple scenarios (Battiston, 2019). In this context, standard approaches to asset pricing are not adequate (Bolton et al., 2020; Monasterolo and Battiston, 2020). In this article, we contribute to fill this relevant knowledge gap showing how it is possible to integrate knowledge of forward-looking climate transition risk in the analysis of portfolio risk. First, we compute the valuation adjustment of corporate bonds based on available knowledge on climate transition scenarios characterised by the introduction of climate policy (i.e. carbon pricing). Information on forward-looking climate policy scenarios aligned with the Paris Agreement climate targets (IPCC, 2014) is provided by climate economic models. Then, we analyse how the probability of default (PD) of a leveraged investor with a portfolio of corporate bonds can be affected by the impact of mild or adverse disorderly transition scenarios characterised by climate policy shocks. Further, we show how financial risk measures (e.g. the Expected Shortfall, ES) depend on key parameters of climate policy shocks and the exposure of the portfolio to low/high-carbon activities. The article is organized as follows. Section 2 describes the model. Section 3 analyses the adjustment in bond's value, PD and spread conditioned to forward-looking transition scenarios. Section 4 analyses the dependence of the ES of a leveraged investor on key parameters of climate policy shock. Section 5 concludes.

2. Model

2.1. Composition of the economy

We consider an economy composed of $n \in \mathbb{N}$ companies, indexed on j that invest in a set $\mathcal{S}$ of sectors of economic activity, indexed on S and characterised by energy technologies (e.g. fossil fuels, renewable energy). A company j that engages in multiple business lines is modelled as a portfolio of activities across sectors S. The company finances its operations issuing corporate bonds. Investors invest in portfolios of corporate bonds. Assessing the climate transition risk exposure of these companies represents a challenge for investors because different activities have different climate transition risk profiles. The concept of carbon-stranded assets (see e.g. Leaton (2012) and van der Ploeg and Rezai (2020)) has provided a powerful metaphor to conceptualize the risks that climate change and a disorderly low-carbon transition could represent for the economy and finance. However, it does not provide a classification of sectors at risk at a level of disaggregation that is relevant for finance and policy.

The classification of economic activities of companies in Climate Policy Relevant Sectors (CPRS) developed by Battiston et al. (2017) allows to address this challenge. CPRS provide a standardized and actionable classification of activities (at the NACE Rev2, 4-digit level[1]) whose revenues could be affected positively or negatively in a disorderly low-carbon transition, based on their energy technology. For this reason, the CPRS classification is considered as a reference for climate financial risk assessment (ESMA, 2020) and has been adopted by several international financial institutions to assess investors' exposure to climate transition risk (EIOPA, 2018; ECB, 2019; Alessi et al., 2019; Battiston et al., 2020).[2]

2.2. Scenarios of climate policies and climate policy shocks

There is growing consensus among financial supervisors and investors that the scenarios of climate policies and regulations are relevant for business performance and for investment decisions (see NGFS (2020)). In this section, we formalise the notions of climate policy scenarios and climate policy shocks used in the model.

- A set of *Climate Policy Scenarios*, describing the future progress of international agreements on climate change mitigation (e.g. with regards to GHG emissions reduction targets compatible with 1.5 and 2 degrees C objectives and the introduction of climate policies. These scenarios are developed by the international scientific community and are reviewed by the Intergovernmental Panel on Climate Change (IPCC,

[1] https://ec.europa.eu/eurostat/documents/3859598/5902521/KS-RA-07-015-EN.PDF

[2] It is worth noting that the level of granularity of CPRS by technology makes it fully compatible with the activities in the EU Taxonomy((PE/20/2020/INIT). The CPRS classification is complementary to the EU Taxonomy: (i) CPRS consider all activities that are relevant for climate transition risk but are not considered by the EU Taxonomy because non sustainable in relation to its climate dimension (e.g. fossil fuels); (ii) CPRS have a characterization of climate financial risk of investments.

2014, 2018). Scenarios are denoted as

$$\text{ClimPolScen} = \{B, P_1, ..., P_l, ..., P_{n^{\text{Scen}}}\}, \tag{1}$$

with B denoting a Base scenario in which no climate policy is introduced, and P_l denoting scenarios in which climate policies are introduced (e.g. carbon pricing).

- A set of *Economic Output Trajectories* for each country C, sector S, scenario P, estimated with a given climate economic model M. These trajectories represent the output of sectors characterised by different energy technologies (i.e. fossil fuels or renewable energy based), conditioned to the P scenarios and consistent with the corresponding GHG emission reduction targets:

$$\text{EconScen} = \{Y_{1,1,1,1}, ..., Y_{C,S,P,M,...}\} \tag{2}$$

  This notion formalises the quantitative knowledge produced by existing climate economic models.[3]

- A set of *Transition Scenarios*, describing a disorderly transition (see more below) from the Base scenario to one of the other climate policy scenarios:

$$\text{TranScen} = \{BP_1, ..., BP_l, ..., BP_{n^{\text{Scen}}}\} \tag{3}$$

- A set of *Climate Policy Shocks* on economic output for country C, sector S, conditional to each Transition Scenario, estimated with a given model M. The shocks are obtained

[3]Among others, Integrated Assessment Models (IAM, Weyant (2017)) provide databases of such trajectories by energy technology, with high degree of granularity, see e.g. McCollum et al. (2018). Such models have been used to inform the policy discussion and the IPCC. However, climate economic models have attracted criticism, e.g. with regard to their treatment of risk and uncertainty in the estimation of economic damage (Stoerk et al., 2018); the representation of climate dynamics inconsistent with the climate science (Dietz et al., 2020); the lack of money (Farmer et al., 2015), of finance and its complexity in mitigation scenarios (Monasterolo, 2020). For instance, trajectories produced by such models do not consider climate tipping points (Lenton et al., 2019).

as differences in the output of individual sectors between the trajectory in the B scenario and the corresponding trajectory in the Climate Policy Scenario P, for the same model M, as described in Eq. 4[4].

$$\text{PolShock} = \{..., \frac{Y_{C,S,P,M} - Y_{C,S,B,M}}{Y_{C,S,B,M}}, ...\} \tag{4}$$

2.3. Orderly and disorderly transition

The transition to a low-carbon economy could occur orderly or disorderly. An *orderly transition* is defined as a situation in which the climate policies (e.g. a carbon tax, carbon pricing) are introduced early and in a coordinated way among countries, allowing investors to anticipate the policy impact on their business. In contrast, a *disorderly transition* is defined as a situation in which investors may not fully anticipate the policy impact on their business, which triggers an adjustment in asset prices, either positive or negative, respectively for low-carbon or high-carbon energy technologies. Reasons for the lack of anticipation include the fact that climate policies are introduced late or suddenly or because of political uncertainty around its implementation (e.g. see the recent case of "Brexit").

In the context of a disorderly transition, issuers in high-carbon activities, whose revenues depend solely on fossil fuel technologies (e.g. coal, oil, gas production), will incur losses from the so-called *carbon-stranded assets*. These losses negatively affect the value of the issuer's financial assets, and can cascade to financial portfolios invested into them (Stolbova et al., 2018). The impact of a climate policy shock can also be positive for issuers with shares of revenues from low-carbon activities (i.e. renewable energy technologies such as solar and wind).

Which transition scenarios will occur is uncertain and endogenous because it depends on governments' chosen path for the introduction of climate policies, and from investors' expectations and reactions, i.e. their *climate sentiments* (Dunz et al., 2020). In order to

[4]Note that in general the shock can be calculated either across trajectories characterised by different energy technology mix and climate policies as in Monasterolo et al. (2018), or across years (e.g. 2020 to 2100, within the same trajectory, as in Battiston et al. (2017).

account for these characteristics when assessing transition risk, investors and managers need to depart from the idea of "most likely/feasible scenario" and consider sets of several, even worst case, scenarios in order to avoid underestimating losses.

2.4. Impact of a disorderly transition on the company's revenues

We consider an issuer j that operates in several CPRS sectors and whose revenues are affected, either negatively or positively, in a Transition Scenario BP, based on their energy technology (respectively, renewable-based positively, fossil fuels-based negatively) We can decompose the net shock on issuer j's revenues as follows:

$$u_j(BP) = \frac{\mathrm{rev}_j(P) - \mathrm{rev}_j(B)}{\mathrm{rev}_j(B)} = \sum_S \left(\frac{\mathrm{rev}_{j,S}(P) - \mathrm{rev}_{j,S}(B)}{\mathrm{rev}_{j,S}(B)} \quad \frac{\mathrm{rev}_{j,S}(B)}{\mathrm{rev}_j(B)} \right)$$

$$u_j(BP) = \sum_S (u_{j,S}(BP) \quad w_{j,S}(B)), \tag{5}$$

where $u_{j,S}(BP)$ denotes the Climate Policy Shock on the revenues of S; $w_{j,S}(B)$ denotes the share on revenues of S. Operatively, shock on j's revenues, $u_{j,S}(BP)$, can be approximated as shocks on output at the level of the corresponding macroeconomic sectors S, which are provided by climate economic models.[5]

While the model allows to analyse all sectors at fine level of granularity, in the following we focus on the following activities: Primary Energy Fossil (**PrFos**), Electricity Fossil (**ElFos**), Renewable (**ElRen**). Eq. 5 becomes:

$$\begin{aligned} u_j(BP) = {} & u_{j,\mathrm{PrFos}}(BP)\, w_{j,\mathrm{PrFos}}(B) + \\ & u_{j,\mathrm{ElFos}}(BP)\, w_{j,\mathrm{ElFos}}(B) + u_{j,\mathrm{ElRen}}(BP)\, w_{j,\mathrm{ElRen}}(B). \end{aligned} \tag{6}$$

The impact of the Transition Scenario BP on the revenues of j, results in a shock $\xi_j(BP)$ in the value of j's assets as follows:

$$\xi_j(BP) = \chi_j^0\, u_j(BP), \tag{7}$$

where χ_j^0 denotes the elasticity of assets with respect to revenues.

[5]Note that we do not need to assume that the negative shock on high-carbon issuers will remain higher in the long run, but only for the period of time that is comparable with the holding time of the investors' portfolio. This means that the negative shock lasts until the maturity of the bond.

2.5. Model for bond valuation

We develop a simple model for counterparty credit valuation, defining the default conditions and the probability of default (PD) in the case of corporate bonds' portfolio. We consider a risky (defaultable) bond of corporate issuer j, issued at t_0 with maturity T. The bond value at time T, with bond *Recovery Rate* R (i.e. % of notional recovered upon default), and *Loss-Given-Default* LGD (i.e. % loss) can be defined as:

$$v_j(T) = \begin{cases} R_j = (1 - \mathrm{LGD}_j) & \text{if j defaults (with prob. } q_j) \\ 1 & \text{else (with prob. } 1 - q_j). \end{cases} \tag{8}$$

The expected value of bond's payoff can be written as:

$$\mathbb{E}[v_j] = (1 - q_j) + q_j\, R_j = 1 - q_j\,(1 - R_j) = 1 - q_j\, \mathrm{LGD}_j. \tag{9}$$

The bond price v_j^* is equal to the bond discounted expected value, with y_f risk-free rate. The price defines implicitly the yield y_j of j (under the risk neutral measure) as follows:

$$v_j^* = e^{-y_f\, T}\, \mathbb{E}[v_j] = e^{-y_f\, T}\,(1 - q_j \mathrm{LGD}_j) = e^{-y_j\, T}. \tag{10}$$

Finally, the bond spread is defined as:

$$s_j = y_j - y_f, \tag{11}$$

with $e^{-s_j\, T} = 1 - q_i\, \mathrm{LGD}_i$.

2.6. Bond default condition

The value of assets in the corporate bond issuer i's balance sheet are denoted as $A_i(t = 0)$, $A_i(T)$, with $t = 0$ being the time of issuance and T the maturity. The liabilities are considered constant and denoted as $L_i(T)$. Using a long-standing approach following Merton (1974), we model the change in asset value as a stochastic process in order to derive a default condition.[6] Here, we consider two types of shocks on the asset side. The first type of shock

[6]For our purposes, here, it it sufficient to consider a discrete time model (in two time steps). The model could be developed in continuous time, but this would not change the results.

is described by the random variable $\eta_j(T) \in \mathbb{R}$ denoting an idiosyncratic shock (e.g. on company j's productivity). The second type of shock is the Climate Policy Shock, conditional to a given Transition Scenario BP, described by the deterministic variable $\xi_j(BP)$ (see Section2.4). Note that, in principle, the Climate Policy Shock $\xi_j(BP)$ and the idiosyncratic shocks η_j on the issuer could be endogenously related. Nevertheless, it is reasonable to assume that frequent small productivity shocks across time and companies could occur in a similar way with or without the Climate Policy Shock. For instance, relative differences in the quality of management and productivity between two oil companies could be unaffected by the occurrence of the Climate Policy Shock. We formalise this intuition by assuming that $\xi_j(BP)$ is a deterministic shock, conditional to the Transition Scenario BP. The company's default occurs when the asset value falls below the liability value:

$$A_j(T) = A_j(0)(1 + \eta_j(T) + \xi_j(BP)) < L_j(T)$$

The default condition then reads:

$$j\text{'s default} \iff \eta_j(T) \leq \theta_j(BP) = L_j(T)/A_j(0) - 1 - \xi_j(T, BP), \tag{12}$$

with $\theta_j(BP)$ denoting the default threshold under the scenario BP. The shock on issuer j's assets $\xi_j(BP)$ can be either positive or negative (given the composition of j: $\xi_j(BP) > -1$), and possibly correlated across issuers j.

3. Climate risk of the bond's issuer: probability of default and spread

The PD $q_j(BP)$ of issuer j, under the Transition Scenario BP, is the probability that the default condition of Eq. 12 is satisfied:

$$q_j(BP) = \mathcal{P}(\eta_j < \theta_j(BP)) = \int_{\eta_{\inf}}^{\theta_j(BP)} \phi(\eta_j)\, d\eta_j, \tag{13}$$

with $\phi(\eta_j)$ denoting the probability distribution of the idiosyncratic shocks η_j, and $\eta_{\inf}$ the lower bound of the distribution support. We also need the following definition.

Definition 1. *The issuer's PD adjustment under the Transition Scenario BP is:*

$$\Delta q_j(BP) = q_j(P) - q_j(B) \tag{14}$$

with $\theta_j(BP) = \theta_j(B) - \xi_j(BP)$.

Further, we assume that the Climate Policy Shock on the company's assets is proportional to the Climate Policy Shock on revenues via an elasticity coefficient, as follows: $\xi_j(BP) = \chi_j^0 \, u_j(BP)$. Under these assumptions, we can characterize how the the Climate Policy Shock impacts on the adjustment in the bond issuer's probability of default (PD), $\Delta q_j(BP)$, in the bond value $\Delta v_j^*(BP)$ and in the bond spread $\Delta s_j(BP)$.[7] We are able to determine the direction of these adjustment as a function of the net shock on revenues that depends on the combination of the issuer's shares of revenues across S. Further, for shocks that are not too large, we provide a linear approximation of the adjustment. The results are formalised in Prop. 1 , Prop. 2, and Prop. 3.[8]

Proposition 1. Consider the following hypotheses: i) the probability distribution of idiosyncratic shocks η_j on the issuer's revenues is independent of the occurrence of the policy shock on company's output $u_j(BP)$; ii) the Cumulative Distribution Function (CDF) of η is a strictly increasing function; iii) the Climate Policy Shock ξ_j on issuer' assets is proportional to the output shock $u_j(BP)$ through the elasticity coefficient $\xi_j = \chi_j^0 \, u_j(BP)$. Conditional to a Transition Scenario BP, the following properties hold for the adjustment in j's PD, $\Delta q_j(BP)$.

(i) $\Delta q_j(BP)$ increases with the net shock magnitude, $|u_j(BP)|$, if $u_j^{BP} < 0$, and decreases viceversa;

(ii) Under the approximation of small shock, i.e. $u_j(BP) << 1$, $\Delta q_j(BP)$ can be linearized to be proportional to the shock on revenues of Climate Policy Relevant Sectors:

$$\Delta q_j(BP) \approx -\chi_j \left(u_{j,\text{PrFos}}(BP) \, w_{j,\text{PrFos}} + u_{j,\text{ElFos}}(BP) \, w_{j,\text{ElFos}} + u_{j,\text{ElRen}}(BP) \, w_{j,\text{ElRen}} \right). \tag{15}$$

[7] Since, for a given model M and country C, the Transition Scenario BP determines the Climate Policy Shock on revenues $u_j(BP)$, we drop the index of country and model in the aforementioned quantities.

[8] The proofs of all the propositions are reported in the Appendix.

Definition 2. *The adjustment in the value of the issuer's bond conditional to the Transition Scenario BP, $\Delta v_j^*(BP)$, is defined as the change in the discounted expected value of the bond, resulting from the Transition Scenario BP on issuer j's revenues $u_j(BP)$:*

$$\Delta v_j^*(BP) = v_j^*(BP) - v_j^*(B). \tag{16}$$

Proposition 2. In line with the model of Section 2.4, assume that, conditional to the Transition Scenario, the Climate Policy Shock on issuer's asset is $\xi_j(BP)$. Consider a zero-coupon corporate bond v_j^*, issued by j. The following properties hold:

(i) The expression of the adjustment of the value of the bond Δv_j^*, conditional to BP reads:

$$\Delta v_j^*(BP) = v_j^*(q_j(BP)) - v_j^*(q_j(B)) = -e^{-y_f\,T}\,\Delta q_j(BP)\mathrm{LGD}_j. \tag{17}$$

(ii) If $u_j(BP) < 0$, then $\Delta v_j^*(BP)$ is negative and increases in absolute value with the policy shock's magnitude $|u_j(BP)|$.

(iii) If $u_j(BP) > 0$ m then $\Delta v_j^*(BP)$ is positive and increases with the policy shock's magnitude, with the constraint $v_j^* \leq 1$.

Definition 3. *The Climate Spread Δs_j is defined as the change in the spread s_j, conditional to the Climate Policy Shock BP:*

$$\Delta s_j = s_j(q_j(P) - s_j(q_j(B)). \tag{18}$$

Proposition 3. Conditional to the climate policy shock scenario, the following hold:

(i) The expression of the climate spread reads:

$$\Delta s_j(BP) = s_j(BP) - s_j(B) = -(1/T)\,(log(v_j^*(BP)) - log(v_j^*(B)) - (y_f(BP) - y_f(B)). \tag{19}$$

(ii) $\Delta s_j(BP)$ increases with the magnitude of the policy shock on revenues $|u_j(BP)|$, if $u_j(BP) < 0$;

(iii) $\Delta s_j(BP)$ decreases with the magnitude of the policy shock on revenues, if $u_j(BP) > 0$;

(iv) Under the approximation of small shock $u_j(BP) << 1$, $\Delta s_j(BP)$ can be linearized to be

$$\Delta s_j(BP) \approx -\frac{1}{T}\chi_j \left(u_{j,\text{PrFos}}\, w_{j,\text{PrFos}} + u_{j,\text{ElFos}}\, w_{j,\text{ElFos}} + u_{j,\text{ElRen}}\, w_{j,\text{ElRen}}\right) \tag{20}$$

4. Climate risk in investor's portfolio

4.1. Dependence of Expected Shortfall and Probability of Default on key parameters

In the previous sections, we have seen how the adjustment in default probability Δq of individual bonds is related to Climate Policy Shocks on the issuer's revenues across low-carbon/high-carbon sectors. In this Section, we investigate how the Climate Policy Shocks, interacting with the other key parameters, impact on the risk for the investor. We consider an investor i with a portfolio of m corporate bonds, financed with leverage. Leverage is defined as the ratio of total asset over equity, $\Lambda = A/E \geq 1$. We denote with $z_i(T)$ the investor i's *portfolio value* and with $\pi_i(T)$ the *portfolio rate of return* at time T, with W_{ij} the amount (numeraire) of j's bonds purchased by i at time t_0. We have:

$$z_i(T) = \sum_j W_{ij} v_j(T), \qquad \pi_i = \frac{z_i(T) - z_i(t_0)}{z_i(t_0)}. \tag{21}$$

The Expected Shortfall (ES) is a standard indicator for risk management that is widely used in the context of financial regulation, in particular for stress-testing of banks (e.g. Basel III Accords). The ES captures the notion of the average worst-case loss, i.e. the loss that occurs above a certain threshold. The ES can be defined in terms of Value-at-Risk and it contributes to overcome its limitations.[9] We adapt the definition of ES and VaR to the context of a Transition Scenario.

Definition 4. ***Climate VaR*** *is the Value-at-Risk of the portfolio of investor i, conditional to Transition Scenario BP with: π portfolio loss, $\psi_P(\pi)$ distribution of losses conditional to the Climate Policy Shock, and α is the confidence level The expression reads:*

$$\int_{Climate\,VaR_\alpha(BP)}^{1} \psi_{BP}(\pi)\, d\pi = \alpha. \tag{22}$$

[9] Contrary to VaR, ES is known to be a *coherent risk measure* and to be more sensitive to events in the tail of the distribution (Acerbi and Tasche, 2002).

Definition 5. *The **Climate ES** is the average of the losses above the Climate VaR:*

$$ES(BP) = \frac{1}{\alpha} \int_0^\alpha ClimateVaR_{\alpha'}(BP)\, d\alpha'. \tag{23}$$

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We now analyse the dependence of ES on key parameters. To derive some analytical results, we consider an equally weighted portfolio of zero-coupon with the same PD, denoted as q, and the same loss-given-default LGD. Some of the results will hold when these assumptions are relaxed. The probability distribution of losses for investor i can be expressed in terms of the joint probability distribution of the bonds' outcomes, which are binary variables described by Eq. 8. If the bonds' outcomes are independent, the joint probability distribution can be expressed in terms of the Binomial distribution and some analytical results can be derived. If instead bonds' outcomes are interdependent (which is the case in many real-world contexts), the problem of writing the probability distribution of gains and losses for investor i is not mathematically tractable[10]. Therefore, we generate multivariate vectors of outcomes from a sample space, by making use of a multivariate Gaussian copula. Each outcome of the sample space corresponds to a vector of payoff across the bonds. By assigning a given correlation matrix, the copula method allows to implement a structure of statistical dependency across bonds' payoff such that the correlations among bonds' payoff across the sample space are those prescribed in the correlation matrix given as input. Here, we consider the simplest dependency structure given by a symmetric correlation matrix with the same value on the off-diagonal elements. The resulting correlation among the default events of pairs of bonds is denoted as ρ.

Figure 1 shows that, as q and ρ increase, the distribution of losses is reshaped and that both VaR and ES move to the right. Figure 2(Left) shows the ES of the investor's portfolio (measured as a fraction of the total initial value of the portfolio) as a function of q, for selected values of the correlation ρ among the bonds in the portfolio. In the absence of

[10]The probability distribution of a sum of random variables is an open mathematical problem, see e.g. (McNeil et al., 2015).

correlation, i.e. with $\rho = 0$, the ES increases almost linearly with q. For value $q > 0.02$, the derivative is smaller than 2 and the curve has a convex shape. For positive values of the correlation $\rho > 0$, the curve of ES is also increasing and shows a concave shape in q. Moreover, ES increases with ρ: curves of PD for higher values of ρ are above those with lower values. Note that, in the case of $\rho \geq 0.1$ and for values of q in range of $q \geq 0.1$ (empirically, the most relevant), the sensitivity of ES with respect to q is larger than 1.

Proposition 4 provides a result on the direction of the impact of adjustments $\Delta q(BP)$ on ES.

Proposition 4. Consider an equally weighted portfolio of zero-coupon bonds with the same PD, q, and with the same loss-given-default LGD. Bonds' default are independent events. Conditional to the Transition Scenario BP, ES(BP) increases with the adjustment on bond default probability $\Delta q(BP)$.

We then investigate how the investor's PD depends on the key parameters. Investor's PD is the probability that losses exceed the equity of the investor. In its general form, the model allows to investigate the investor's PD for any arbitrary portfolio of bonds with varying weights across bonds and varying shares of revenues from low/high-carbon activities across issuers. Under the above assumptions of homogeneity across bonds, the investor's PD is the probability that the number of bonds in default exceeds a critical number. This simplifies the analysis while providing the relevant insights. Figure 2 shows how the investor's PD depends on the bonds' PD q for selected values of the correlation ρ among the bonds in the portfolio. In the absence of correlation, i.e. with $\rho = 0$, the PD increases non-linearly with q and the curve has an s-shape, with a steep transition around 0.05, which corresponds to an expected fraction of bonds in default equal to the critical loss that causes the default of the investor. For positive values of the correlation $\rho > 0$, the curve of PD is also increasing and its s-shaped is stretched. Moreover, PD increases with ρ: curves of PD for higher values of ρ are above those with lower values.Note that for values $q > 0.02$ and $\rho = 0.2$, the derivative of PD w.r.t. q is about 7 implying a strong sensitivity of PD on q.

The results for the case of independent defaults ($\rho = 0$) are formalized in Prop. 5.

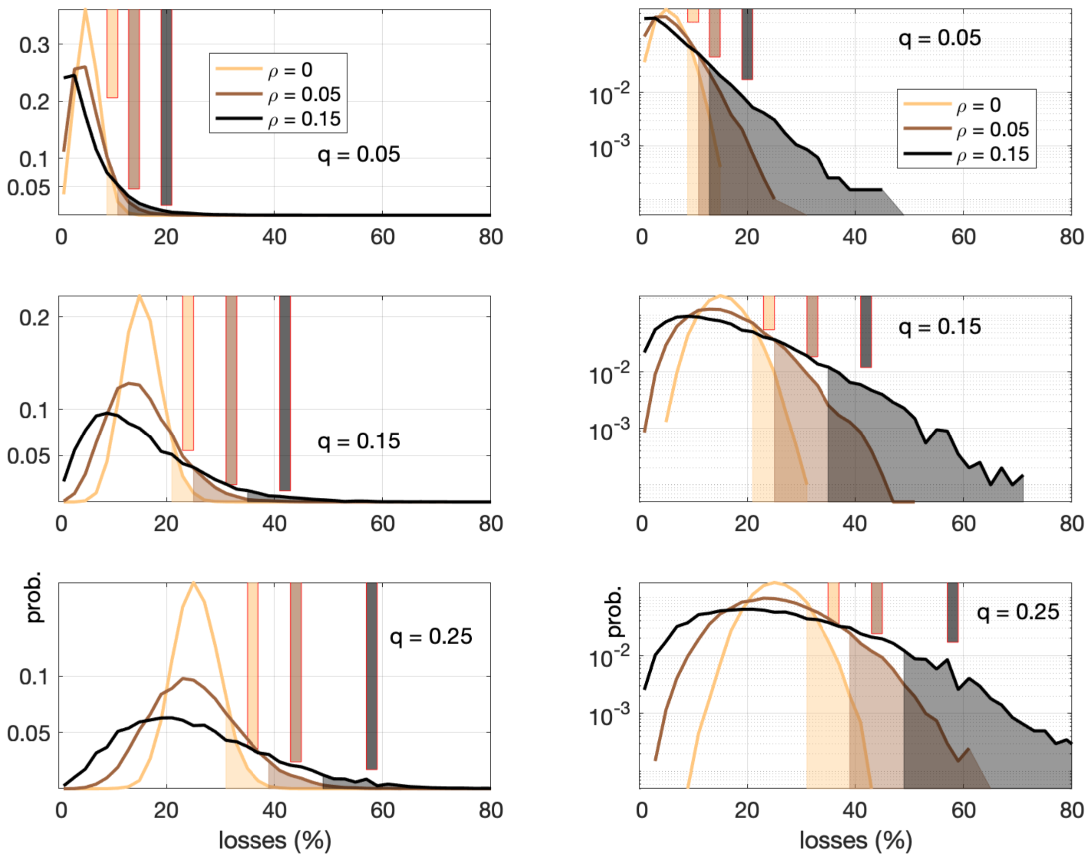

Figure 1: Probability distribution of losses on an example portfolio of $m = 100$ bonds, in percentage of the total portfolio value, for varying levels of bond default probability q and bond default correlation ρ. Bond default correlation is modelled by means of a Gaussian copula with symmetric dependence matrix (see text) **Left panels**: distribution of losses in linear scale, for selected values of individual bond PD q, increasing from top to bottom panel. In each panel, three selected level of correlation ρ are shown with color code indicated in the legend. The area plots indicate the right tail of the distribution of the losses exceeding the 95% VaR. The vertical bars (in color codes corresponding to ρ levels) indicate the position on the x-axis of the value of the ES of the distribution of losses. **Right panels**: same as left panels but in log scale in order to show the details of the tail of the distribution of losses.

Proposition 5. Consider a leveraged investor with an equally weighted portfolio, of zero-coupon bonds, with issuers having independent defaults occurring with the same probability q and with the loss-given-default LGD. The following properties hold:

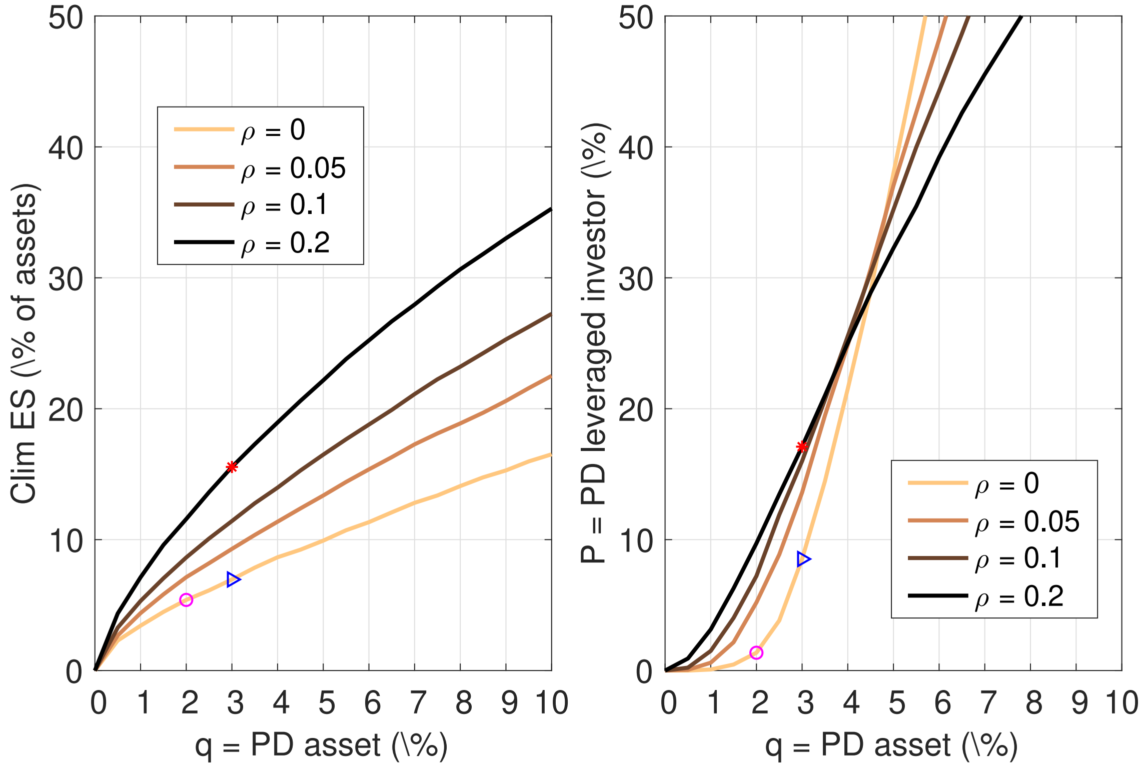

Figure 2: ES and PD of an investor holding an example portfolio of $m = 100$ bonds, exposed to Climate Policy Shocks. (Left) Dependence of ES (measured as percentage of total portfolio value) as a function of the bond default probability q and bond default correlation ρ. Markers refer to the following (q, ES) points: (0.02, 0.054) (circle); (0.03, 0.07) (triangle);(0.03, 0.16) (star). (Right) Dependence of PD as a function of q and ρ as in the left panel. Markers refer to the following (q, PD) points: (0.02, 0.014) (circle); (0.03, 0.085) (triangle);(0.03, 0.17) (star).

(i) The investor's PD, $P(m, \Lambda, q)$ can be expressed in terms of the binomial distribution $\mathcal{B}(m^*_-, m, q)$:

$$P(m, \Lambda, q) = \mathcal{P}(X \geq m^*_-) = 1 - \mathcal{B}(m^*_-, m, q) \tag{24}$$

(ii) The investor's PD is non decreasing in: a) the investor's leverage Λ; b) the loss-given-default LGD; c) the bond default probability q.

Under a further homogeneity assumption we can also derive how the PD depends explicitly on the share of low/high-carbon revenues of the issuers in the portfolio. An example result is stated in Prop. 6.

Proposition 6. Consider an equally weighted portfolio of zero-coupon bonds with the same PD, q, and the same loss-given-default LGD. Assume further that: a) all issuers j have the same shares of revenues across the three sectors Primary Energy Fossil, Electricity Fossil, Electricity Renewable: $w_{j,\text{PrFos}}(B), w_{j,\text{ElFos}}(B), w_{j,\text{ElRen}}(B)$; b) the Transition Scenario is such that, for all j , $u_{j,\text{PrFos}}(BP) < 0$, $u_{j,\text{ElFos}}(BP) < 0$, $u_{j,\text{ElRen}}(BP) > 0$ and the net shock on revenues $u_j(BP) < 0$. Conditional to the Transition Scenario BP, we have:

(i) Then, ES(BP) decreases with the share of revenues $w_{j,\text{ElRen}}(B)$.

(ii) Then, PD(BP) decreases with the share of revenues $w_{j,\text{ElRen}}(B)$.

Figure 2(right) illustrates also the impact of uncertainty on the two key parameters. Consider the value of investor's PD estimated assuming $q = 0.02$ and $\rho = 0$ (purple circle). If, because of an estimation error, it turned out that $q = 0.03$ (blue triangle), the PD would be about 8 times larger. Further, if it turned out that $\rho = 0.2$ (red star), PD would be about 15 times larger than its estimate. The results highlight the importance of considering the uncertainty on climate transition scenarios. Indeed, small changes in bond PD and correlation can imply large changes in the investor's PD. Hence, investors financial stability is highly sensitive to the climate transition scenarios.

4.2. Investor's ES and PD with multiple transition scenarios

If multiple shock scenarios can occur and their probabilities p_l of occurrence are known, then the investor's PD can be computed as the expected value of the investor's PD across scenarios. We investigate how the probability of occurrence of different disorderly Transition Scenarios BP affect the investor's PD. For the sake of simplicity, we consider the case of two scenarios (i.e. Mild, Adverse) that are mutually exclusive, i.e. $p_M = 1 - p_A$. The results can be extended to the case of more than two scenarios with similar implications. In the Mild Scenario, bonds' PD and correlation are low: ($q = 0.01$, $\rho = 0.01$). In the Adverse scenario, bonds' PD varies in [01] and and the correlation is high (e.g. $\rho = 0.3$). Figure 3 shows the dependence of ES and PD of the investor on the bond PD q, for varying levels of probability of occurrence of the adverse scenario. In the right panel, consider point 1 (purple circle)

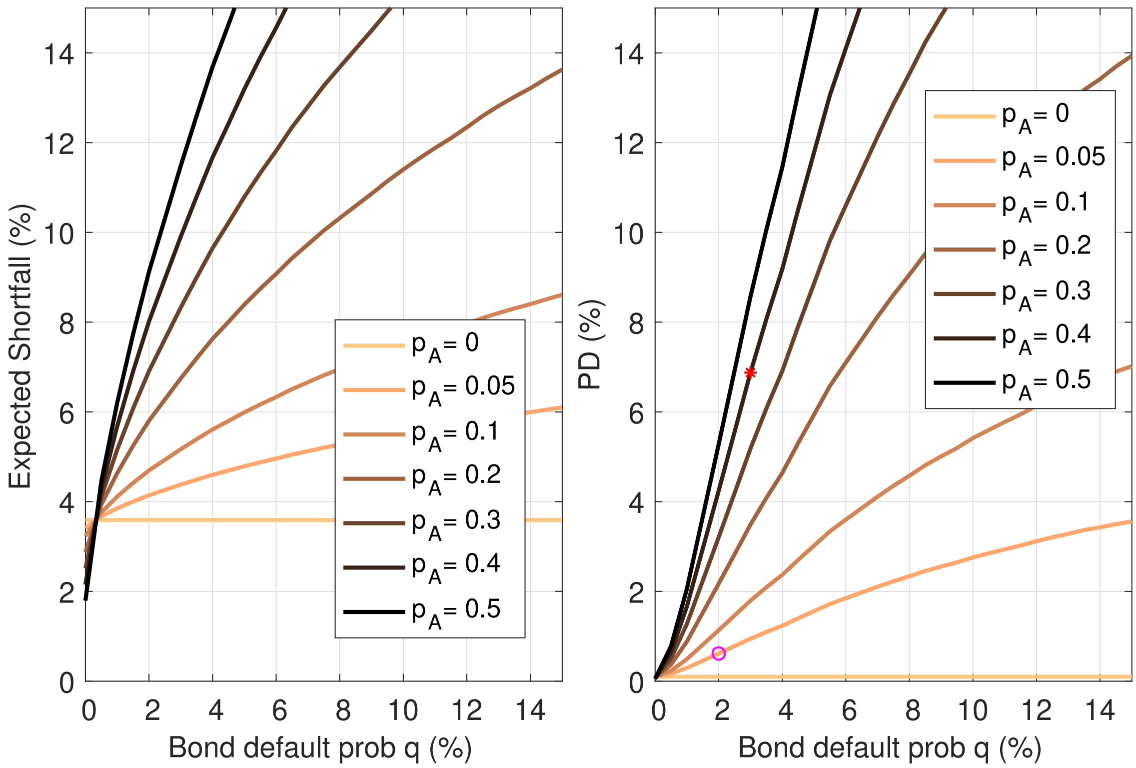

Figure 3: ES and PD of the investor versus bond probability of default q, for varying levels of probability of occurrence of the Adverse Scenario.

with $q = 0.02, p_A = 0.05$ and PD of 0.007, and point 2 (red star) with $q = 0.03, p_A = 0.4$, and PD about 0.07. Thus, an error in the severity of the disorderly transition scenario and its probability of occurrence implies a possible error of a factor 10 in the PD.

In reality, the probabilities p_l of occurrence of the various possible scenarios are endogenous. Indeed, the the perception of climate related financial risk by investors impacts on their investment choices and thus on the very occurrence of the climate transition scenario. This situation leads to a circular relation between the vector of p_l and itself $p_l = f(p_l)$, which in general, could have no solution or multiple solutions, resulting in wide variations of p_l. By shedding light on how the investor's PD depends on the scenarios probabilities, our model can inform future work on this issue.

5. Conclusion

This paper provides the first model to assess how forward-looking climate transition risk scenarios and companies' shares of revenues across low/high-carbon activities affect the valuation adjustment of corporate bonds, as well as the adjustments of an investor's Expected Shortfall and PD. Our model allows to consider the impact of the uncertainty of climate transition risk on risk metrics that are relevant for risk management. By applying analytical work and numerical computations, we show that the ES and the PD of a leveraged investor increase with the impact of the climate policy shock on bond issuers' revenues (in low/high-carbon activities). Further, the PD of a leveraged investor is sensitive to small change in the PD of the bond and thus to the selection of climate transition scenarios. Moreover, assumptions on the set of climate transition scenarios and their probability of occurrence plays a main role for investors' risk management. Therefore, limiting the underestimation of losses due to climate transition risk, requires central banks to design climate stress tests with a wide enough sets of scenarios.

The model provides an operative, reference framework for central banks (e.g. those involved in the NGFS) applicable with several types of climate economic and macroeconomic models, contributing to open new research directions in climate finance. In particular, follow up work include investigating portfolio optimization strategy in the face of climate transition risk, the model calibration and the treatment of endogenous probabilities of default.

Appendix I - Proofs of propositions

Proof. Proof of Prop.1 (i) By assumption, the probability distribution of the idiosynchratic shocks is the same in the B and P scenarios. However, there is a different threshold in the default condition. The adjustment in PD, $\Delta q_j(BP)$, is by definition the difference in PD in the two scenarios, i.e. a difference of two integrals with the same integrand function φ, but computed on two different intervals:

$$\Delta q_j(BP) = q_j(BP) - q_j(B) = \int_{\theta_j(BP)}^{\theta_j(P)} \phi(\eta_j)\, d\eta_j. \tag{25}$$

Consider the B scenario fixed and hence the default threshold $\theta_j(B)$. Then, Eq. 25 implies that the PD adjustment $\Delta q_j(BP)$ is an increasing function of $\theta_j(BP)$ for $\theta_j(P) > \theta_j(B)$. It is also decreasing with decreasing values of $\theta_j(P)$, under the constraints $q_j(P) \geq 0$ and $\theta_j(P) > \theta_j(B)$. In turn, Eq.12 implies that $\theta_j(P)$ is strictly increasing (decreasing) in $\xi_j(BP)$ if $\xi_j(BP)$ is negative (positive). Further, $\xi_j(BP)$ is strictly increasing with the net shock $u_j(BP)$ if $u_j(BP) > 0$ and viceversa. This completes the proof of item (i).

(ii) We simplify the notation as $u_j = u_j(BP)$. Recall that $\theta_j(BP)$ is a function of $u_j(BP)$ and thus $q_j(BP)$ and $\Delta q_j(BP)$ are functions of $u_j(BP)$. We can linearize the function $q_j(u_j(BP))$ as a deviation from $q_j(B)$. This holds exactly in the limit of $u_j^{BP} \to 0$ i.e. where scenario P is very close to scenario B. For finite deviations, it holds that $q_j(BP) \approx q_j(B) + \frac{dq_j(BP)}{du_j}((u_j - 0)$. It follows that $\Delta q_j(BP) = q_j(P) - q_j(B) \approx \frac{dq_j(BP)}{du_j}((u_j - 0)$. We then apply the formula for the derivative of an integral dependent by parameter. We have:

$$\Delta q_j(P) = \frac{d(\int_{\eta_{\inf}}^{\theta_j(BP)} \phi(\eta_j)\, d\eta_j)}{du_j}\,(u_j - 0) = \varphi(\theta_j(BP))\frac{d\theta_j(BP)}{du_j}(u_j - 0), \qquad (26)$$

because $\eta_{\inf}$ is a constant. Recall that $\theta_j(BP) = L_j(T)/A_j(t_0) - 1 - \xi_j(T, P)$ and that $\xi_j(T, P) = \chi u_j(BP)$. It follows that $\Delta q_j(BP) \approx -\varphi(\theta_j(BP))\, \chi_j^0(BP)\, u_j$. We then introduce the sensitivity coefficient $\chi_j(BP) = \varphi(\theta_j(BP))\, \chi_j^0(BP)$ and we obtain $\Delta q_j(BP) \approx -\chi_j(BP)\, u_j$. Finally, by substituting u_j in the last expression with its expression from Eq. 6 we obtain the statement of item (ii). □ □

Proof. Proof of Prop. 2 (i) Recall that the expression of the value of the bond reads $v_j^* = e^{-y_f\,T}\,\mathbb{E}[v_j] = e^{-y_f\,T}\,(1 - q_j\mathrm{LGD}_j)$. It follows that

$$v_j^*(q_j(BP) - v_j^*(q_j(B)) = e^{-y_f\,T}\,\mathbb{E}[v_j] = e^{-y_f\,T}\,(-)(q_j(BP) - q_j(B))\mathrm{LGD}_j), \qquad (27)$$

which proves item (i). The proofs of items (ii) and (iii) follow directly from item (i) above and item (i) of Prop. 1. □ □

Proof. Proof of Prop. 3 (i) From the definition of bond spread and yield it follows: $s_j(BP) = y_j(BP) - y_f(BP)$, with $y_j = -(1/T)\, log(v_j^*(BP))$, and $s_j(B) = y_j(B) - y_f(B)$, with $y_j(B) =$

$-(1/T)\,log(v_j^*(B))$. The definition of climate spread implies $\Delta s_j(BP) = s_j(BP) - s_j(B) = -(1/T)\,(log(v_j^*(BP)) - log(v_j^*(B)) - (y_f(BP) - y_f(B))$. The proofs of items (ii-iii) follow from item (i) and from items (ii-iii) of Prop. 2. (iv) For small climate policy shock, we neglect the difference in the risk free rate in the two scenarios and we linearize the expression of item (i) in the neighborhood of $u_j(BP) = 0$. We have: $log(v_j^*(BP)) \approx 1 - v_j^*(BP)$ and $log(v_j^*(B)) \approx 1 - v_j^*(B)$. This implies $\Delta s_j(BP) \approx -(1/T)\Delta v_j(BP)$. The proof then follows from substituting the expressions obtained in item (i) of this proposition and item (iii) of Prop. 1. □ □

Proof. Proof of Prop. 4 Losses on the bond portfolio equal the product of the LGD times the number m_- of defaulting bonds out of the m bonds in the portfolio. If bonds' defaults are independent and have the same probability of occurrence q, then m_- follows the Binomial distribution, $\mathcal{B}(m_-, m, q)$. Now we make use of a result on stochastic dominance of the binomial distribution (Klenke and Mattner, 2010). : $q_1 \geq q_2 \Longrightarrow \mathcal{B}(m_-, m, q_1)\ \leq \mathcal{B}(m_-, m, q_2)\,\forall m_-$. Equivalently, $\mathcal{B}(m_-, m, q_1)$ "dominates" $\mathcal{B}(m, q_2$ in the sense of first order stochastic dominance, denoted as $\mathcal{B}(m, q_1)\ \leq \mathcal{B}(m, q_2$ (with the symbol "$\leq$" applied to the distributions). For a continuous and strictly monotonic distribution function such as the Binomial, the VaR at the confidence level α is the α-quantile of the probability distribution, i.e. the inverse of the distribution function at the point α. Since $\mathcal{B}(m_-, m, q_1)\ \leq \mathcal{B}(m_-, m, q_2)\,\forall m_-$ implies that $\mathcal{B}_{q_1}^{-1}(\alpha\ \geq \mathcal{B}_{q_2}^{-1}(\alpha\,$, it follows that the VaR is non-decreasing with default probability q. It follows that also the ES, which is the average of the VaR across confidence levels, is non-decreasing in the default probability q. □ □

Proof. Proof of Prop. 5 (i) The PD of a leveraged investor, with leverage Λ_i is the probability that the relative loss on the investor's assets is a fraction larger than $\frac{1}{\Lambda_i}$. Indeed, by definition of leverage, a loss of $A_i \frac{1}{\Lambda_i}$ equals $A_i \frac{E_i}{A_i} = E_i$ and a loss larger than equity implies default. A loss on the bond portfolio equals the product of the LGD times the fraction $\frac{m_-}{m}$ of defaulting bonds in the portfolio. Hence, PD equals the following probability:

$$\mathcal{P}(\frac{m_-}{m}\,\mathrm{LGD}) > \frac{1}{\Lambda}) = \mathcal{P}(m_- > m_-^*), \tag{28}$$

where $m_-^* = [\frac{m}{\mathrm{LGD}\,\Lambda}]$ with the bracket denoting the integer part. Since bonds' defaults are independent, this is also the complement to 1 of the cumulative binomial distribution $\mathcal{B}(m_-^*, m, q)$. Thus, the agent default probability, $P(m, \Lambda, q)$ reads:

$$\begin{aligned} P(m,\Lambda,q) &= \mathcal{P}(X \geq m_-^*) = 1 - \mathcal{P}(X < m_-^*) = \\ &= 1 - \mathcal{B}(m_-^*, m, q) = 1 - \sum_{k=\{0,1,\ldots,m_-^*\}} \frac{m!}{k!(m-k)!} q^k (1-q)^{m-k}. \end{aligned} \tag{29}$$

(iia) As leverage Λ increases, $P_i(m, \Lambda, q)$ can only increase because the sum (in the right hand side of the second line of the equation) is taken over a smaller set (up to $\frac{m}{\Lambda}]$), it comes with a minus sign, and all terms in the sum are positive. (iib) The same dependence holds for LGD, thus proving the statement. (iic) We make use of results on stochastic dominance as in the Proof of Prop. 4. The stochastic dominance relation, together with Eq. 29 implies:

$$q_1 \geq q_2 \Longrightarrow P(m, \Lambda, q_1) \leq P(m, \Lambda, q_2), \tag{30}$$

i.e. the default probability is non decreasing with the parameter q. □ □

Proof. Proof of Prop. 6 (i) Under the specified assumptions, all issuers have the same $\Delta q(BP)$, which, by virtue of Prop. 1 is positive, because the shock on revenues is negative. Further, the assumption on the Transition Scenario implies that an increase in the share of revenues from the Renewable sector, makes the net shock on revenues less negative and thus reduces $\Delta q(BP)$. As a consequence of Prop.4, item (i), then ES is also reduced. (ii) The proof is the same as for item (i) except that in the conclusion, we use the relation between investor PD and $\Delta q(BP)$ from Prop.5. □ □

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